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# Deep Sparse Rectifier Neural Networks

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## Abstract

While logistic sigmoid neurons are more biologically plausible than hyperbolic tangent neurons, the latter work better for training multi-layer neural networks. This paper shows that rectifying neurons are an even better model of biological neurons and yield equal or better performance than hyperbolic tangent networks in spite of the hard non-linearity and non-differentiability at zero, creating sparse representations with true zeros, which seem remarkably suitable for naturally sparse data. Even though they can take advantage of semi-supervised setups with extra-unlabeled data, deep rectifier networks can reach their best performance without requiring any unsupervised pre-training on purely supervised tasks with large labeled datasets. Hence, these results can be seen as a new milestone in the attempts at understanding the difficulty in training deep but purely supervised neural networks, and closing the performance gap between neural networks learnt with and without unsupervised pre-training.

## 1 Introduction

Many differences exist between the neural network models used by machine learning researchers and those used by computational neuroscientists. This is in part

because the objective of the former is to obtain computationally efficient learners, that generalize well to new examples, whereas the objective of the latter is to abstract out neuroscientific data while obtaining explanations of the principles involved, providing predictions and guidance for future biological experiments. Areas where both objectives coincide are therefore particularly worthy of investigation, pointing towards computationally motivated principles of operation in the brain that can also enhance research in artificial intelligence. In this paper we show that two common gaps between computational neuroscience models and machine learning neural network models can be bridged by using the following linear by part activation :  $\max(0, x)$ , called the rectifier (or hinge) activation function. Experimental results will show engaging training behavior of this activation function, especially for *deep architectures* (see Bengio (2009) for a review), i.e., where the number of hidden layers in the neural network is 3 or more.

Recent theoretical and empirical work in statistical machine learning has demonstrated the importance of learning algorithms for deep architectures. This is in part inspired by observations of the mammalian visual cortex, which consists of a chain of processing elements, each of which is associated with a different representation of the raw visual input. This is particularly clear in the primate visual system (Serre *et al.*, 2007), with its sequence of processing stages: detection of edges, primitive shapes, and moving up to gradually more complex visual shapes. Interestingly, it was found that the features learned in deep architectures resemble those observed in the first two of these stages (in areas V1 and V2 of visual cortex) (Lee *et al.*, 2008), and that they become increasingly invariant to factors of variation (such as camera movement) in higher layers (Goodfellow *et al.*, 2009).

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Regarding the training of deep networks, something that can be considered a breakthrough happened in 2006, with the introduction of Deep Belief Networks (Hinton *et al.*, 2006), and more generally the idea of initializing each layer by unsupervised learning (Bengio *et al.*, 2007; Ranzato *et al.*, 2007). Some authors have tried to understand why this unsupervised procedure helps (Erhan *et al.*, 2010) while others investigated why the original training procedure for deep neural networks failed (Bengio and Glorot, 2010). From the machine learning point of view, this paper brings additional results in these lines of investigation.

We propose to explore the use of rectifying nonlinearities as alternatives to the hyperbolic tangent or sigmoid in deep artificial neural networks, in addition to using an  $L_1$  regularizer on the activation values to promote sparsity and prevent potential numerical problems with unbounded activation. Nair and Hinton (2010) present promising results of the influence of such units in the context of Restricted Boltzmann Machines compared to logistic sigmoid activations on image classification tasks. Our work extends this for the case of pre-training using denoising autoencoders (Vincent *et al.*, 2008) and provides an extensive empirical comparison of the rectifying activation function against the hyperbolic tangent on image classification benchmarks as well as an original derivation for the text application of sentiment analysis.

Our experiments on image and text data indicate that training proceeds better when the artificial neurons are either off or operating mostly in a linear regime. Surprisingly, rectifying activation allows deep networks to achieve their best performance without unsupervised pre-training. Hence, our work proposes a new contribution to the trend of understanding and merging the performance gap between deep networks learnt with and without unsupervised pre-training (Erhan *et al.*, 2010; Bengio and Glorot, 2010). Still, rectifier networks can benefit from unsupervised pre-training in the context of semi-supervised learning where large amounts of unlabeled data are provided. Furthermore, as rectifier units naturally lead to sparse networks and are closer to biological neurons' responses in their main operating regime, this work also bridges (in part) a machine learning / neuroscience gap in terms of activation function and sparsity.

This paper is organized as follows. Section 2 presents some neuroscience and machine learning background which inspired this work. Section 3 introduces rectifier neurons and explains their potential benefits and drawbacks in deep networks. Then we propose an experimental study with empirical results on image recognition in Section 4.1 and sentiment analysis in Section 4.2. Section 5 presents our conclusions.

## 2 Background

### 2.1 Neuroscience Observations

For models of biological neurons, the activation function is the expected firing rate as a function of the total input currently arising out of incoming signals at synapses (Dayan and Abott, 2001). An activation function is termed, respectively *antisymmetric* or *symmetric* when its response to the opposite of a strongly excitatory input pattern is respectively a strongly inhibitory or excitatory one, and *one-sided* when this response is zero. The main gaps that we wish to consider between computational neuroscience models and machine learning models are the following:

- Studies on brain energy expense suggest that neurons encode information in a sparse and distributed way (Attwell and Laughlin, 2001), estimating the percentage of neurons active at the same time to be between 1 and 4% (Lennie, 2003). This corresponds to a trade-off between richness of representation and small action potential energy expenditure. Without additional regularization, such as an  $L_1$  penalty, ordinary feedforward neural nets do not have this property. For example, the sigmoid activation has a steady state regime around  $\frac{1}{2}$ , therefore, after initializing with small weights, all neurons fire at half their saturation regime. This is biologically implausible *and* hurts gradient-based optimization (LeCun *et al.*, 1998; Bengio and Glorot, 2010).
- Important divergences between biological and machine learning models concern non-linear activation functions. A common biological model of neuron, the leaky integrate-and-fire (or *LIF*) (Dayan and Abott, 2001), gives the following relation between the firing rate and the input current, illustrated in Figure 1 (left):

$$f(I) = \begin{cases} \left[ \tau \log \left( \frac{E+RI-V_r}{E+RI-V_{th}} \right) + t_{ref} \right]^{-1}, & \text{if } E + RI > V_{th} \\ 0, & \text{if } E + RI \leq V_{th} \end{cases}$$

where  $t_{ref}$  is the refractory period (minimal time between two action potentials),  $I$  the input current,  $V_r$  the resting potential and  $V_{th}$  the threshold potential (with  $V_{th} > V_r$ ), and  $R$ ,  $E$ ,  $\tau$  the membrane resistance, potential and time constant. The most commonly used activation functions in the deep learning and neural networks literature are the standard *logistic sigmoid* and the

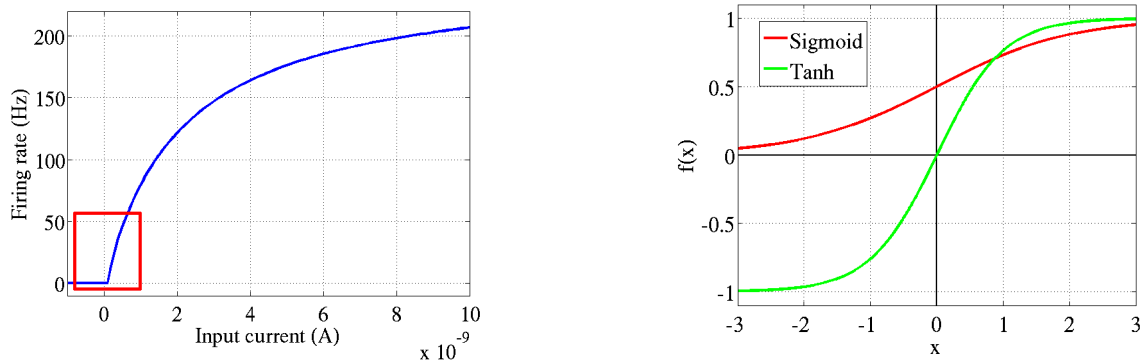


Figure 1: *Left: Common neural activation function motivated by biological data. Right: Commonly used activation functions* in neural networks literature: logistic sigmoid and hyperbolic tangent (*tanh*).

*hyperbolic tangent* (see Figure 1, right), which are equivalent up to a linear transformation. The hyperbolic tangent has a steady state at 0, and is therefore preferred from the optimization standpoint (LeCun *et al.*, 1998; Bengio and Glorot, 2010), but it forces an antisymmetry around 0 which is absent in biological neurons.

## 2.2 Advantages of Sparsity

Sparsity has become a concept of interest, not only in computational neuroscience and machine learning but also in statistics and signal processing (Candes and Tao, 2005). It was first introduced in computational neuroscience in the context of sparse coding in the visual system (Olshausen and Field, 1997). It has been a key element of deep convolutional networks exploiting a variant of auto-encoders (Ranzato *et al.*, 2007, 2008; Mairal *et al.*, 2009) with a sparse distributed representation, and has also become a key ingredient in Deep Belief Networks (Lee *et al.*, 2008). A sparsity penalty has been used in several computational neuroscience (Olshausen and Field, 1997; Doi *et al.*, 2006) and machine learning models (Lee *et al.*, 2007; Mairal *et al.*, 2009), in particular for deep architectures (Lee *et al.*, 2008; Ranzato *et al.*, 2007, 2008). However, in the latter, the neurons end up taking small but non-zero activation or firing probability. We show here that using a rectifying non-linearity gives rise to real zeros of activations and thus truly sparse representations. From a computational point of view, such representations are appealing for the following reasons:

- **Information disentangling.** One of the claimed objectives of deep learning algorithms (Bengio, 2009) is to disentangle the factors explaining the variations in the data. A dense representation is highly entangled because almost any change in the input modifies most of

the entries in the representation vector. Instead, if a representation is both sparse and robust to small input changes, the set of non-zero features is almost always roughly conserved by small changes of the input.

- **Efficient variable-size representation.** Different inputs may contain different amounts of information and would be more conveniently represented using a variable-size data-structure, which is common in computer representations of information. *Varying the number of active neurons allows a model to control the effective dimensionality of the representation for a given input and the required precision.*
- **Linear separability.** Sparse representations are also more likely to be linearly separable, or more easily separable with less non-linear machinery, simply because the information is represented in a high-dimensional space. Besides, this can reflect the original data format. In text-related applications for instance, the original raw data is already very sparse (see Section 4.2).
- **Distributed but sparse.** Dense distributed representations are the richest representations, being potentially exponentially more efficient than purely local ones (Bengio, 2009). Sparse representations' efficiency is still exponentially greater, with the power of the exponent being the number of non-zero features. They may represent a good trade-off with respect to the above criteria.

Nevertheless, forcing too much sparsity may hurt predictive performance for an equal number of neurons, because it reduces the effective capacity of the model.

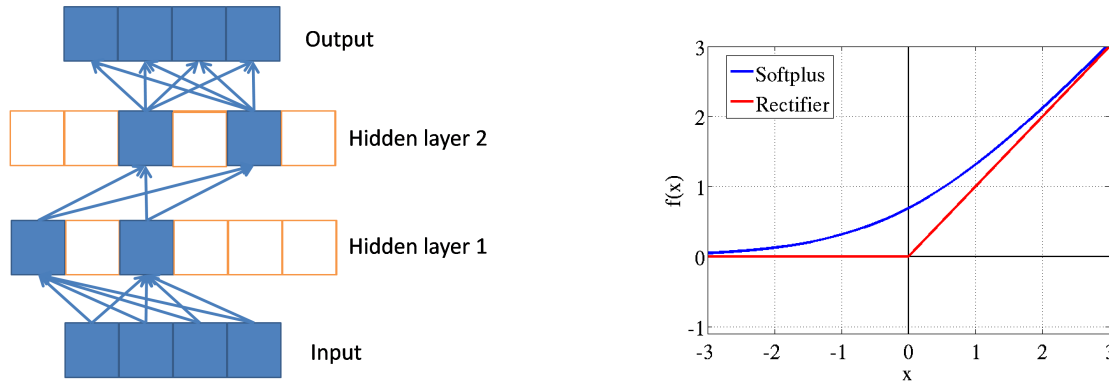


Figure 2: *Left: Sparse propagation of activations and gradients in a network of rectifier units.* The input selects a subset of active neurons and computation is linear in this subset. *Right: Rectifier and softplus activation functions.* The second one is a smooth version of the first.

### 3 Deep Rectifier Networks

#### 3.1 Rectifier Neurons

The neuroscience literature (Bush and Sejnowski, 1995; Douglas and al., 2003) indicates that *cortical neurons are rarely in their maximum saturation regime*, and suggests that their activation function can be approximated by a rectifier. Most previous studies of neural networks involving a rectifying activation function concern recurrent networks (Salinas and Abbott, 1996; Hahnloser, 1998).

The rectifier function  $\text{rectifier}(x) = \max(0, x)$  is one-sided and therefore does not enforce a sign symmetry<sup>1</sup> or antisymmetry<sup>1</sup>: instead, the response to the opposite of an excitatory input pattern is 0 (no response). However, we can obtain symmetry or antisymmetry by combining two rectifier units sharing parameters.

**Advantages** The rectifier activation function allows a network to easily obtain sparse representations. For example, after uniform initialization of the weights, around 50% of hidden units continuous output values are real zeros, and this fraction can easily increase with sparsity-inducing regularization. Apart from being more biologically plausible, sparsity also leads to mathematical advantages (see previous section).

As illustrated in Figure 2 (left), the only non-linearity in the network comes from the path selection associated with individual neurons being active or not. For a given input *only a subset of neurons are active*. Computation is *linear* on this subset: once this subset of neurons is selected, the output is a linear function of

the input (although a large enough change can trigger a discrete change of the active set of neurons). The function computed by each neuron or by the network output in terms of the network input is thus linear by parts. We can see the model as an *exponential number of linear models that share parameters* (Nair and Hinton, 2010). Because of this linearity, gradients flow well on the active paths of neurons (there is no gradient vanishing effect due to activation non-linearities of sigmoid or tanh units), and mathematical investigation is easier. Computations are also cheaper: there is no need for computing the exponential function in activations, and sparsity can be exploited.

**Potential Problems** One may hypothesize that the hard saturation at 0 may hurt optimization by blocking gradient back-propagation. To evaluate the potential impact of this effect we also investigate the softplus activation:  $\text{softplus}(x) = \log(1 + e^x)$  (Dugas *et al.*, 2001), a smooth version of the rectifying non-linearity. We lose the exact sparsity, but may hope to gain easier training. However, experimental results (see Section 4.1) tend to contradict that hypothesis, suggesting that hard zeros can actually help supervised training. We hypothesize that the hard non-linearities do not hurt *so long as the gradient can propagate along some paths*, i.e., that some of the hidden units in each layer are non-zero. With the *credit and blame assigned to these ON units* rather than distributed more evenly, we hypothesize that optimization is easier. Another problem could arise due to the unbounded behavior of the activations; one may thus want to use a regularizer to prevent potential numerical problems. Therefore, we use the  $L_1$  penalty on the activation values, which also promotes additional sparsity. Also recall that, in order to efficiently represent symmetric/antisymmetric behavior in the data, a rectifier network would need

<sup>1</sup>The hyperbolic tangent absolute value non-linearity  $|\tanh(x)|$  used by Jarrett *et al.* (2009) enforces sign symmetry. A  $\tanh(x)$  non-linearity enforces sign antisymmetry.

twice as many hidden units as a network of symmetric/antisymmetric activation functions.

Finally, rectifier networks are subject to ill-conditioning of the parametrization. Biases and weights can be scaled in different (and consistent) ways while preserving the same overall network function. More precisely, consider for each layer of depth  $i$  of the network a scalar  $\alpha_i$ , and scaling the parameters as  $\mathbf{W}'_i = \frac{\mathbf{W}_i}{\alpha_i}$  and  $\mathbf{b}'_i = \frac{\mathbf{b}_i}{\prod_{j=1}^i \alpha_j}$ . The output units values then change as follow:  $\mathbf{s}' = \frac{\mathbf{s}}{\prod_{j=1}^n \alpha_j}$ . Therefore, as long as  $\prod_{j=1}^n \alpha_j$  is 1, the network function is identical.

### 3.2 Unsupervised Pre-training

This paper is particularly inspired by the sparse representations learned in the context of auto-encoder variants, as they have been found to be very useful in training deep architectures (Bengio, 2009), especially for unsupervised pre-training of neural networks (Erhan *et al.*, 2010).

Nonetheless, certain difficulties arise when one wants to introduce rectifier activations into stacked denoising auto-encoders (Vincent *et al.*, 2008). First, the hard saturation below the threshold of the rectifier function is not suited for the reconstruction units. Indeed, whenever the network happens to reconstruct a zero in place of a non-zero target, the reconstruction unit can not backpropagate any gradient.<sup>2</sup> Second, the unbounded behavior of the rectifier activation also needs to be taken into account. In the following, we denote  $\tilde{x}$  the corrupted version of the input  $x$ ,  $\sigma(\cdot)$  the logistic sigmoid function and  $\theta$  the model parameters ( $W_{enc}, b_{enc}, W_{dec}, b_{dec}$ ), and define the linear reconstruction function as:

$$f(x, \theta) = W_{dec} \max(W_{enc}x + b_{enc}, 0) + b_{dec} .$$

Here are the several strategies we have experimented:

1. Use a softplus activation function for the reconstruction layer, along with a quadratic cost:

$$L(x, \theta) = \|x - \log(1 + \exp(f(\tilde{x}, \theta)))\|^2 .$$

2. Scale the rectifier activation values coming from the previous encoding layer to bound them between 0 and 1, then use a sigmoid activation function for the reconstruction layer, along with a cross-entropy reconstruction cost.

$$L(x, \theta) = -x \log(\sigma(f(\tilde{x}, \theta))) - (1 - x) \log(1 - \sigma(f(\tilde{x}, \theta))) .$$

<sup>2</sup>Why is this not a problem for hidden layers too? we hypothesize that it is because gradients can still flow through the active (non-zero), possibly helping rather than hurting the assignment of credit.

3. Use a linear activation function for the reconstruction layer, along with a quadratic cost. We tried to use input unit values either before or after the rectifier non-linearity as reconstruction targets. (For the first layer, raw inputs are directly used.)
4. Use a rectifier activation function for the reconstruction layer, along with a quadratic cost.

The first strategy has proven to yield better generalization on image data and the second one on text data. Consequently, the following experimental study presents results using those two.

## 4 Experimental Study

This section discusses our empirical evaluation of rectifier units for deep networks. We first compare them to hyperbolic tangent and softplus activations on image benchmarks with and without pre-training, and then apply them to the text task of sentiment analysis.

### 4.1 Image Recognition

**Experimental setup** We considered the image datasets detailed below. Each of them has a training set (for tuning parameters), a validation set (for tuning hyper-parameters) and a test set (for reporting generalization performance). They are presented according to their number of training/validation/test examples, their respective image sizes, as well as their number of classes:

- MNIST (LeCun *et al.*, 1998): 50k/10k/10k,  $28 \times 28$  digit images, 10 classes.
- CIFAR10 (Krizhevsky and Hinton, 2009): 50k/5k/5k,  $32 \times 32 \times 3$  RGB images, 10 classes.
- NISTP: 81,920k/80k/20k,  $32 \times 32$  character images from the NIST database 19, with randomized distortions (Bengio and al, 2010), 62 classes. This dataset is much larger and more difficult than the original NIST (Grother, 1995).
- NORB: 233,172/58,428/58,320, taken from Jittered-Cluttered NORB (LeCun *et al.*, 2004). Stereo-pair images of toys on a cluttered background, 6 classes. The data has been preprocessed similarly to (Nair and Hinton, 2010): we subsampled the original  $2 \times 108 \times 108$  stereo-pair images to  $2 \times 32 \times 32$  and scaled linearly the image in the range  $[-1,1]$ . We followed the procedure used by Nair and Hinton (2010) to create the validation set.

Table 1: **Test error on networks of depth 3.** Bold results represent statistical equivalence between similar experiments, with and without pre-training, under the null hypothesis of the pairwise test with  $p = 0.05$ .

Neuron	MNIST	CIFAR10	NISTP	NORB
<b>With unsupervised pre-training</b>				
Rectifier	<b>1.20%</b>	<b>49.96%</b>	<b>32.86%</b>	<b>16.46%</b>
Tanh	<b>1.16%</b>	<b>50.79%</b>	35.89%	17.66%
Softplus	<b>1.17%</b>	<b>49.52%</b>	<b>33.27%</b>	19.19%
<b>Without unsupervised pre-training</b>				
Rectifier	<b>1.43%</b>	<b>50.86%</b>	<b>32.64%</b>	<b>16.40%</b>
Tanh	1.57%	52.62%	36.46%	19.29%
Softplus	1.77%	53.20%	35.48%	17.68%

For all experiments except on the NORB data (LeCun *et al.*, 2004), the models we used are stacked denoising auto-encoders (Vincent *et al.*, 2008) with three hidden layers and 1000 units per layer. The architecture of Nair and Hinton (2010) has been used on NORB: two hidden layers with respectively 4000 and 2000 units. We used a cross-entropy reconstruction cost for tanh networks and a quadratic cost over a softplus reconstruction layer for the rectifier and softplus networks. We chose masking noise as the corruption process: each pixel has a probability of 0.25 of being artificially set to 0. The unsupervised learning rate is constant, and the following values have been explored:  $\{.1, .01, .001, .0001\}$ . We select the model with the lowest reconstruction error. For the supervised fine-tuning we chose a constant learning rate in the same range as the unsupervised learning rate with respect to the supervised validation error. The training cost is the negative log likelihood  $-\log P(\text{correct class}|\text{input})$  where the probabilities are obtained from the output layer (which implements a softmax logistic regression). We used stochastic gradient descent with mini-batches of size 10 for both unsupervised and supervised training phases.

To take into account the potential problem of rectifier units not being symmetric around 0, we use a variant of the activation function for which half of the units output values are multiplied by -1. This serves to cancel out the mean activation value for each layer and can be interpreted either as inhibitory neurons or simply as a way to equalize activations numerically. Additionally, an  $L_1$  penalty on the activations with a coefficient of 0.001 was added to the cost function during pre-training and fine-tuning in order to increase the amount of sparsity in the learned representations.

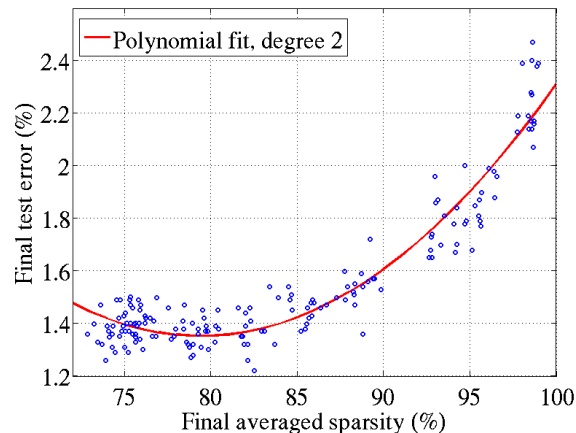


Figure 3: **Influence of final sparsity on accuracy.** 200 randomly initialized deep rectifier networks were trained on MNIST with various  $L_1$  penalties (from 0 to 0.01) to obtain different sparsity levels. Results show that enforcing sparsity of the activation does not hurt final performance until around 85% of true zeros.

comparing all the neuron types<sup>3</sup> on all the datasets, with or without unsupervised pre-training. In the latter case, the supervised training phase has been carried out using the same experimental setup as the one described above for fine-tuning. The main observations we make are the following:

- Despite the hard threshold at 0, networks trained with the rectifier activation function can find local minima of greater or equal quality than those obtained with its smooth counterpart, the softplus. On NORB, we tested a rescaled version of the softplus defined by  $\frac{1}{\alpha} \text{softplus}(\alpha x)$ , which allows to interpolate in a smooth manner between the softplus ( $\alpha = 1$ ) and the rectifier ( $\alpha = \infty$ ). We obtained the following  $\alpha$ /test error couples: 1/17.68%, 1.3/17.53%, 2/16.9%, 3/16.66%, 6/16.54%,  $\infty$ /16.40%. There is no trade-off between those activation functions. Rectifiers are not only biologically plausible, they are also computationally efficient.
- There is almost no improvement when using unsupervised pre-training with rectifier activations, contrary to what is experienced using tanh or softplus. Purely supervised rectifier networks remain competitive on all 4 datasets, even against the pretrained tanh or softplus models.

<sup>3</sup>We also tested a rescaled version of the LIF and  $\max(\text{tanh}(x), 0)$  as activation functions. We obtained worse generalization performance than those of Table 1, and chose not to report them.

**Main results** Table 1 summarizes the results on networks of 3 hidden layers of 1000 hidden units each,

- Rectifier networks are truly deep sparse networks. There is an average exact sparsity (fraction of zeros) of the hidden layers of 83.4% on MNIST, 72.0% on CIFAR10, 68.0% on NISTP and 73.8% on NORB. Figure 3 provides a better understanding of the influence of sparsity. It displays the MNIST test error of deep rectifier networks (without pre-training) according to different average sparsity obtained by varying the  $L_1$  penalty on the activations. Networks appear to be quite robust to it as models with 70% to almost 85% of true zeros can achieve similar performances.

With labeled data, deep rectifier networks appear to be attractive models. They are biologically credible, and, compared to their standard counterparts, do not seem to depend as much on unsupervised pre-training, while ultimately yielding sparse representations.

This last conclusion is slightly different from those reported in (Nair and Hinton, 2010) in which is demonstrated that unsupervised pre-training with Restricted Boltzmann Machines and using rectifier units is beneficial. In particular, the paper reports that pre-trained rectified Deep Belief Networks can achieve a test error on NORB below 16%. However, we believe that our results are compatible with those: we extend the experimental framework to a different kind of models (stacked denoising auto-encoders) and different datasets (on which conclusions seem to be different). Furthermore, note that our rectified model without pre-training on NORB is very competitive (16.4% error) and outperforms the 17.6% error of the non-pretrained model from Nair and Hinton (2010), which is basically what we find with the non-pretrained soft-plus units (17.68% error).

**Semi-supervised setting** Figure 4 presents results of semi-supervised experiments conducted on the NORB dataset. We vary the percentage of the original labeled training set which is used for the supervised training phase of the rectifier and hyperbolic tangent networks and evaluate the effect of the unsupervised pre-training (using the whole training set, unlabeled). Confirming conclusions of Erhan *et al.* (2010), the network with hyperbolic tangent activations improves with unsupervised pre-training for any labeled set size (even when all the training set is labeled).

However, the picture changes with rectifying activations. In semi-supervised setups (with few labeled data), the pre-training is highly beneficial. But the more the labeled set grows, the closer the models with and without pre-training. Eventually, when all available data is labeled, the two models achieve identical performance. Rectifier networks can maximally exploit labeled and unlabeled information.

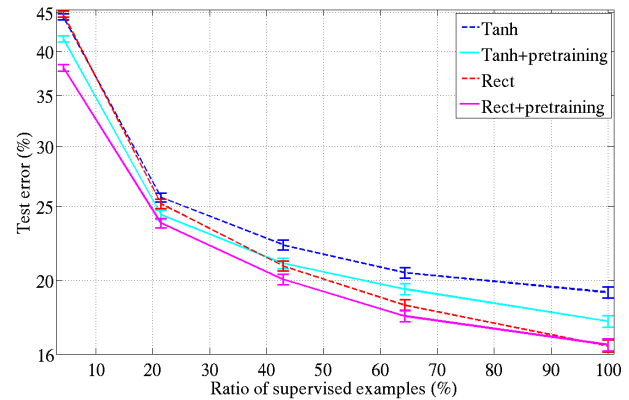


Figure 4: **Effect of unsupervised pre-training.** On NORB, we compare hyperbolic tangent and rectifier networks, with or without unsupervised pre-training, and fine-tune only on subsets of increasing size of the training set.

## 4.2 Sentiment Analysis

Nair and Hinton (2010) also demonstrated that rectifier units were efficient for image-related tasks. They mentioned the intensity equivariance property (i.e. without bias parameters the network function is linearly variant to intensity changes in the input) as argument to explain this observation. This would suggest that rectifying activation is mostly useful to image data. In this section, we investigate on a different modality to cast a fresh light on rectifier units.

A recent study (Zhou *et al.*, 2010) shows that Deep Belief Networks with binary units are competitive with the state-of-the-art methods for sentiment analysis. This indicates that deep learning is appropriate to this text task which seems therefore ideal to observe the behavior of rectifier units on a different modality, and provide a data point towards the hypothesis that rectifier nets are particularly appropriate for sparse input vectors, such as found in NLP. Sentiment analysis is a text mining area which aims to determine the judgment of a writer with respect to a given topic (see (Pang and Lee, 2008) for a review). The basic task consists in classifying the polarity of reviews either by predicting whether the expressed opinions are positive or negative, or by assigning them star ratings on either 3, 4 or 5 star scales.

Following a task originally proposed by Snyder and Barzilay (2007), our data consists of restaurant reviews which have been extracted from the restaurant review site [www.opentable.com](http://www.opentable.com). We have access to 10,000 labeled and 300,000 unlabeled training reviews, while the test set contains 10,000 examples. The goal is to predict the rating on a 5 star scale and performance is evaluated using Root Mean Squared Error (RMSE).<sup>4</sup>

<sup>4</sup>Even though our tasks are identical, our database is

<b>Customer's review:</b>	<b>Rating</b>
"Overpriced, food small portions, not well described on menu."	★
"Food quality was good, but way too many flavors and textures going on in every single dish. Didn't quite all go together."	★★
"Calamari was lightly fried and not oily—good job—they need to learn how to make desserts better as ours was frozen."	★★★
"The food was wonderful, the service was excellent and it was a very vibrant scene. Only complaint would be that it was a bit noisy."	★★★★
"We had a great time there for Mother's Day. Our server was great! Attentive, funny and really took care of us!"	★★★★★

Figure 5: **Examples of restaurant reviews from `www.opentable.com` dataset.** The learner must predict the related rating on a 5 star scale (right column).

Figure 5 displays some samples of the dataset. The review text is treated as a bag of words and transformed into binary vectors encoding the presence/absence of terms. For computational reasons, only the 5000 most frequent terms of the vocabulary are kept in the feature set.<sup>5</sup> The resulting preprocessed data is very sparse: 0.6% of non-zero features on average. Unsupervised pre-training of the networks employs both labeled and unlabeled training reviews while the supervised fine-tuning phase is carried out by 10-fold cross-validation on the labeled training examples.

The model are stacked denoising auto-encoders, with 1 or 3 hidden layers of 5000 hidden units and rectifier or tanh activation, which are trained in a greedy layer-wise fashion. Predicted ratings are defined by the expected star value computed using multiclass (multinomial, softmax) logistic regression output probabilities. For rectifier networks, when a new layer is stacked, activation values of the previous layer are scaled within the interval  $[0,1]$  and a sigmoid reconstruction layer with a cross-entropy cost is used. We also add an  $L_1$  penalty to the cost during pre-training and fine-tuning. Because of the binary input, we use a "salt and pepper noise" (i.e. masking some inputs by zeros and others by ones) for unsupervised training of the first layer. A zero masking (as in (Vincent *et al.*, 2008)) is used for the higher layers. We selected the noise level based on the classification performance, other hyperparameters are selected according to the reconstruction error.

Table 2: **Test RMSE and sparsity level** obtained by 10-fold cross-validation on OpenTable data.

Network	RMSE	Sparsity
No hidden layer	0.885 ± 0.006	99.4% ± 0.0
Rectifier (1-layer)	0.807 ± 0.004	28.9% ± 0.2
Rectifier (3-layers)	<b>0.746</b> ± 0.004	53.9% ± 0.7
Tanh (3-layers)	0.774 ± 0.008	00.0% ± 0.0

Results are displayed in Table 2. Interestingly, the RMSE significantly decreases as we add hidden layers to the rectifier neural net. These experiments confirm that rectifier networks improve after an unsupervised pre-training phase in a semi-supervised setting: with no pre-training, the 3-layers model can not ob-

much larger than the one of (Snyder and Barzilay, 2007).  
<sup>5</sup>Preliminary experiments suggested that larger vocabulary sizes did not markedly change results.

tain a RMSE lower than 0.833. Additionally, although we can not replicate the original very high degree of sparsity of the training data, the 3-layers network can still attain an overall sparsity of more than 50%. Finally, on data with these particular properties (binary, high sparsity), the 3-layers network with tanh activation function (which has been learnt with the exact same pre-training+fine-tuning setup) is clearly outperformed. The sparse behavior of the deep rectifier network seems particularly suitable in this case, because the raw input is very sparse and *varies in its number of non-zeros*. The latter can also be achieved with sparse internal representations, not with dense ones.

Since no result has ever been published on the OpenTable data, we applied our model on the Amazon sentiment analysis benchmark (Blitzer *et al.*, 2007) in order to assess the quality of our network with respect to literature methods. This dataset proposes reviews of 4 kinds of Amazon products, for which the polarity (positive or negative) must be predicted. We followed the experimental setup defined by Zhou *et al.* (2010). In their paper, the best model achieves a test accuracy of 73.72% (on average over the 4 kinds of products) where our 3-layers rectifier network obtains 78.95%.

## 5 Conclusion

Sparsity and neurons operating mostly in a linear regime can be brought together in more biologically plausible deep neural networks. Rectifier units help to bridge the gap between unsupervised pre-training and no pre-training, which suggests that they may help in finding better minima during training. This finding has been verified for four image classification datasets of different scales and all this in spite of their inherent problems, such as zeros in the gradient, or ill-conditioning of the parametrization. Rather sparse networks are obtained (from 50 to 80% sparsity for the best generalizing models, whereas the brain is hypothesized to have 95% to 99% sparsity), which may explain some of the benefit of using rectifiers.

Furthermore, rectifier activation functions have shown to be remarkably adapted to sentiment analysis, a text-based task with a very large degree of data sparsity. This promising result tends to indicate that deep sparse rectifier networks are not only beneficial to image classification tasks and might yield powerful text mining tools in the future.



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